



**Modelling Mountain Pine Beetle
in the Chilcotin using the
Westwide Pine Beetle Model**

Canadian Forest Service

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March 29, 2004

Citation: **Beukema, S.J. and D.C.E. Robinson.** 2004. Modelling Mountain Pine in the Chilcotin using the Westwide Pine Beetle Model. Prepared by ESSA Technologies Ltd., Vancouver, BC for the Canadian Forest Service, Victoria, BC. 21 pp.

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1.0 Introduction

The mountain pine beetle (MPB) outbreak currently ongoing in BC is having a huge impact on lodgepole stands in the province. Several questions are being asked about the outbreak and its aftermath. Models such as MPBSim (based on Safranyik et al. 1999) and MPB-Seles are being used to predict the short term (10-year) location and impacts of the outbreak. Both models, however, are designed to look primarily at outbreak conditions. Neither looks at the long term impacts on stand development or other processes such as *Ips* mortality or the potential change in risk of fire.

One such model that has the potential to look at these issues is the Westwide Pine Beetle Model (WWPBM, Beukema et al. 1994, 1997), which simulates the impact of mountain pine beetle (*Dendroctonus ponderosae Hopkins*), western pine beetle (*Dendroctonus brevicomis LeConte*), and *Ips* spp. on western pine tree species. The model is a multi-stand, landscape-level model and operates when linked to the Forest Vegetation Simulator stand projection model (FVS, Stage 1973) and to the FVS Parallel Processing Extension (PPE, Crookston and Stage 1991). BC already has a variant of FVS called Prognosis^{BC} which has been designed to work well in complex, multi-species, multi-age stands (Robinson 2000). It can simulate the changes in the structure and composition of a stand with or without the mortality caused by MPB. The other two components, the WWPBM and the PPE, are currently in use in the US. As part of this project, prototype versions of them were created for BC and linked to Prognosis^{BC}.

The WWPBM simulates the contagion of beetles in a landscape, moving beetles between stands based on the current location of the beetles, the relative attractiveness of the stands in the landscape, and the assumed state of the area surrounding the simulated landscape. Once beetles have been allocated to a stand, the model simulates the attack of trees within the stand based on the size, species, and vigour of the trees. Beetle levels in a stand in the following year are related to the amount of beetle-related mortality that occurred in the stand. The model is designed to simulate both epidemic and endemic conditions, and therefore, the transition between the two states.

The purpose of this project was to use the prototype WWPBM on a local landscape as the first step in the evaluation of its utility in BC for modelling MPB and *Ips* impacts and the subsequent development of the stand. This report describes the results of that project and suggests some future steps that may help in the evaluation process.

2. Methods

Study Area

This analysis was conducted on a subset of the Chilcotin Plateau, a region around Tatla Lake. The area was chosen because of many of the 30 stands for which there are data are concentrated near that area. The study area is a 149,000 ha landscape containing 6904 stands ranging in size from less than 1.0m² to 659ha.

For each stand, we had information about its size, location, age, site characteristics, and dominant vegetation. Only 11 of the stands contained sample plots. These were spread throughout the landscape (Figure 2.1).

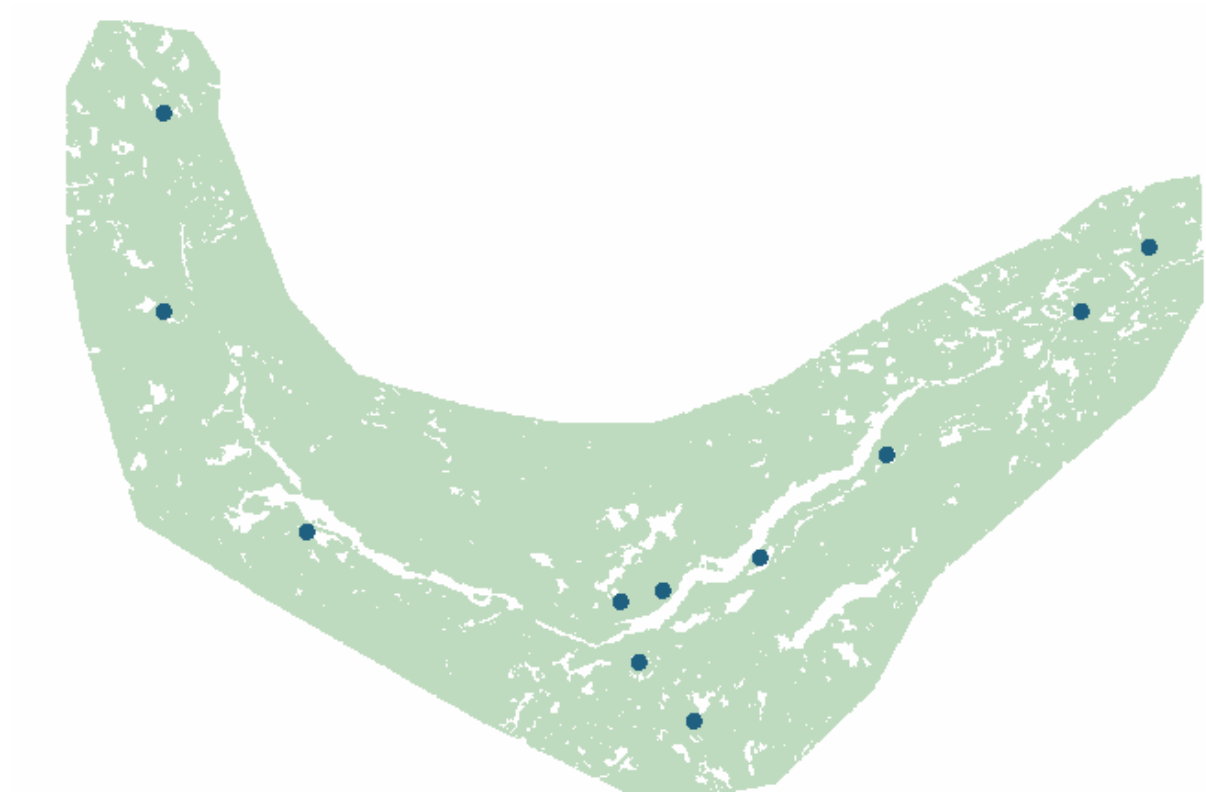


Figure 2.1: Location of the 11 sample plots within the study area.

Tree Lists

Prognosis^{BC} needs information about the trees in every stand that will be simulated by the model. These are entered into the model in a “tree list” which contains information about the species, size (at a minimum dbh for most trees and height for regenerating trees), and number of trees per hectare with those characteristics.

Tree information was provided for 30 stands in the Chilcotin. These stands were not necessarily inside the study area, but most were either inside or nearby. Sample information was taken in 1987 and included live and dead trees, saplings, and regeneration.

For this project, we needed to have treelists that replicated the 1976 conditions. Some simple rules were used.

1. Sapling-size trees were changed to smaller trees. All saplings with a dbh of 5.5cm were given a new dbh of 4.5cm. Smaller saplings were given different sizes based on their species, ranging from 1.3 to 1.7cm dbh (Table 2.1). These values were found by experimenting with two stands. The dbh would be set to some value, the stands would be run through Prognosis^{BC}, and then the size of the trees would be examined in 1987. The values chosen were the ones for which the projected young trees in 1987 were close in size to the 1987 data values. There will be variation in growth between stands, especially after accounting for beetle mortality, but these values were still considered better than choosing values at random.
2. The sampled regeneration was assumed to occur in 1987, and so was not present in the initial treelists. This regeneration was added to the stands using the PLANT keyword with the specified species, densities, and height classes, and assuming 100% survival.
3. All other live trees were assumed to be the same height and dbh that they were in 1987. This will overestimate the size of the trees, but since the trees in the Chilcotin grow very slowly (see Growth section), the overestimate is likely not significant.
4. All dead trees were assumed to be alive at the size given in the sample data.

Table 2.1: Dbh to use in the 1976 treelist for saplings smaller than 5.5 cm in the 1987 data.

Species	New dbh
Aspen	1.7 cm
Douglas-fir	1.5 cm
All others (mostly Lodgepole)	1.3 cm

Only eleven of the 6,904 stands in the study area contained treelists, but in order to run the model, we needed treelists for all stands. Because the landscape is relatively homogeneous, we decide to assign the treelists to the other stands randomly, but with some rules to ensure that gross errors did not occur. The rules were as follows:

1. If the target stand is one of the eleven stands with data, assign the associated treelist.
6. If the target stand is over 180 years old, assign that stand to the oldest treelist (stand 113).
3. For all other stands, pick one of the 30 treelists at random.
4. If the age of the target stand is less than 12 years old, then the stand has been harvested since 1976 and we do not know the original age of the stand. Thus, just assign the picked treelist.
5. Otherwise, compare the age of the picked treelist to the age of the target stand.
6. If the ages of the two differs by more than 30 years, then pick another stand and repeat until the age difference is less than 30 years.

When doing this tree list assignment, we did not look at any other stand characteristics, such as slope, aspect, or elevation. This could be done in later applications.

Growth Modifiers

Most stands being simulated by the model were in the SBPSxc, which is outside the calibrated range of Prognosis. Thus, three main changes were made to stand characteristics or to the default growth parameters in the model to accommodate this.

BEC

Most stands in the Chilcotin landscape are designated as being in the SBPSxc, with a small number of IDFdk4. No site series information was available for the stands, so our best default assumption was to use a mesic (/01) site.

Prognosis^{BC} Version 3.0 has not yet been calibrated for any variants of the SBS or SBPS. The best available growth model inside Prognosis^{BC} is probably the IDFdk4, which grows PL using equations developed by pooling all sample plots with a Site Index of 18 or less. This mapping to IDFdk4 is also supported by the most recent Site Index assignments for the SBPSxc (see www.for.gov.bc.ca/hre/sibec/reports/sisu2003ByUnit.xls), which assign a site index of 12 to PL from the IDFdk4/01 and 15 from the SBPSxc/01. If the stands in our simulation are indeed mesic, then we may be underestimating the growth of the SBPSxc stands by assigning them to the IDFdk4. Other growth modifiers that we added (see below) may however mitigate any errors.

Growth Multipliers

Some stands contained information about the 10-year growth increment of individual trees. Prognosis^{BC} can use this information to scale its predicted large-tree growth, a big asset when being used in areas for which it is not calibrated. We did an analysis of the scale factors that would be expected based on the growth increment data.

Five of the 15 stands grew lodgepole pine better than the default (scale factors greater than 1.0), but two thirds were lower (Table 2.2). The median and mean are 0.52 and 0.75 respectively. Since growth increments are not available for the other 15 stands, we decided that as a first approximation, it would be simpler to treat all stands equally and assign them all the median scale factor, 0.52, as a growth modifier. An alternative approach would be to retain the scale factors for those stands that had calibration data, and use the median value for the others. Another would be to choose randomly selected multipliers from the observed set. But each alternative carries its own set of assumptions and neither really provides any insight into the variation in growth rates. The impact of variation in growth rate (in the absence of other disturbances) is uncertain without further study and the development of a version of Prognosis^{BC} calibrated for the SBPS.

Table 2.2: Scale factors predicted for lodgepole pine by Prognosis^{BC} for 15 stands in the Chilcotin, based on the growth increment data of selected trees. The “n” is the number of tree records in the stand that contain growth increment data.

Stand	n	scale
103	12	0.45
104	16	1.07
105	15	0.36
107	12	1.06
113	7	0.47
116	14	0.36
118	12	0.39
119	11	2.03
121	11	1.33
124	16	0.67
125	11	0.52
126	5	0.48
128	6	1.01
129	16	0.21
130	7	0.90

Maximum Basal Area

A key driver of mortality in Prognosis^{BC} is the defined maximum basal area of the stand. For the Chilcotin, we asked a local expert (D. Conly, Lignum. pers. comm) for some estimates, and based on that information we chose a maximum basal area of 25 m²/ha.

Initial MPB Conditions

The model needs to be initialised with a list of the stands that contained beetle-killed trees, and the size and density of those trees. The data listed approximately 65 stands with severe MPB mortality in the year prior to our simulation (Figure 2.2). No information was available about initial *Ips* levels. We calculated the initial density of host trees in the 65 stands, and assumed that the equivalent of 50% of the stems greater than 15cm were infested with MPB, and 50% of the stems less than 15cm were infested with *Ips*. If these values are too high, then the model outbreak may progress faster than seen in reality.

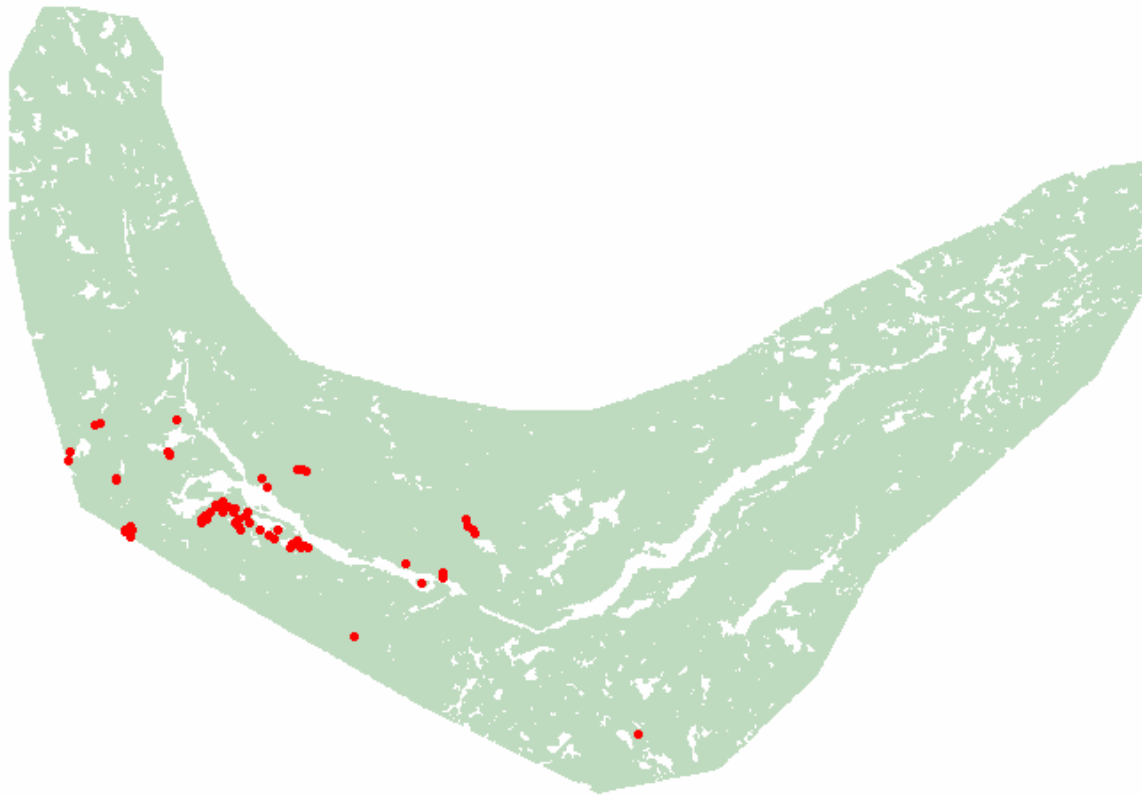


Figure 2.2: Location of the stands which contained the initial MPB and *Ips*.

Other Model Inputs

The goal of these simulations was to change as little as possible from the basic model behaviour. From this, we can judge the strengths and weaknesses of the model. Some weaknesses may later be solved by altering some of the parameters in the model (either in code or through keywords), while others may be inherent.

Beetle parameters

Some key parameters that were changed, however, were in basic beetle biology. Based on expert opinion (T. Shore and L. Safranyk, pers. comm.) we altered the distance that a beetle will travel, and the reproduction curve for MPB. We set the maximum distance that a beetle will travel by choice to 4 km (down from the default value of 24 km). The lower end of the reproduction curve stayed the same as in the base model, which assumes that at 15cm, the amount of beetle coming out of a tree is equal to what went into the tree. The upper value was changed to assume that at 35 cm, six times as many beetles come out of a tree as went into the tree. This is significantly higher than the default value which assumes that a maximum of four times as many beetles emerge from trees 91 cm and larger. In both cases, the reproductive values are linearly interpolated between the minimum and maximum values.

One scenario attempted to simulate the severe winter of 1985 which killed many of the beetles. We used the assumption that in 1985, reproduction would be only 10% of normal conditions.

Note that none of the parameters for *Ips* were altered in any way.

Outside world

The model can be simulated with or without accounting for the fact that the study area is in the middle of a larger landscape. We did scenarios with two different assumptions. In one scenario, we ignored the outside world and assumed that the study area was acting in isolation. Because of the short distance that the beetles fly and the large size of the landscape, this assumption would have less impact than in a smaller landscape. Two other simulations were done assuming that the world outside the study area looked like the study area. Thus, as the average conditions in the study area changed, so did the average conditions in the world outside. Because the Chilcotin is relatively homogeneous, this is a reasonable assumption.

There is a third way of simulating the model, and that is to explicitly tell the model in any given year about the conditions of the outside world. This option works well if the conditions are well known, and if they are significantly different from those inside the landscape. For example, if the outside world was known to have experienced a severe fire or a large amount of harvesting, it would no longer look like the study area. Alternatively, the boundaries of the study area could have been drawn based on large geographic boundaries (the ocean or a grassland for example), or a lack of host type. In these simulations, however, we did not have enough knowledge to guide any choice of how the conditions outside the study area might be changing, and so we did not do any simulations using this option.

Harvesting

In the data for 1987, some stands were less than 12 years old. These were stands that, for the most part, had been harvested between 1976 and 1987. In the affected stands, we thus added a harvesting event that would take out all trees except regenerating trees (if any).

Spatial Information

The PPE and WWPBM need to know about the size, location, and neighbours of each stand. This information was not provided directly by the client, although the client did provide an ArcView map. We wrote scripts in ArcView which calculated this information and printed it to files in the appropriate format for use by the PPE.

Scenarios

Three different scenarios were simulated (Table 2.3). One ran just the basic simulation. The other two assumed the outside world was present, and one of these also included the kill event. The model contains no facility to start beetles in the landscape during a simulation (other than forcing conditions using the outside world), so the other suggested scenarios could not be done.

Table 2.3: Summary of the characteristics of the three scenarios simulated using the WWPBM.

Run #	Name	Outside world	Kill event
1	Base	No	No
2	Outside world	Average	No
3	Kill event	Average	Yes

All runs were simulated from 1976 to 2036 years. All timesteps were 5 years, except one 6-year timestep to enable us to get output in 1987, and one four-year timestep to enable us to get output in 2001. Output in these years could then be compared to the 1987 and 2001 data.

3. Results and Discussion

The three scenarios showed very different behaviours. There are two ways to look at the results: at the level of the landscape or at the behaviour of selected stands.

Landscape-level Results

The average beetle population in the landscape shows that the assumptions that we make about whether or not to include the outside world in our simulations, and whether we account for the 1985 severe winter have a big impact on the predicted dynamics (Figure 3.1). In the base case, with no outside world, the model predicts that the outbreak would peak in 1991. With an outside world, the outbreak would grow more slowly, and would peak between 2001 and 2016. The winter kill event delays this outbreak to around 2030. Note that this is for the entire landscape. Individual stands may contain much higher levels (Figure 3.2), or may not have any beetles. Figure 3.2 also shows that the build-up of beetles in individual stands after the winter kill is starting to become significant in some stands between 2001 and 2006.

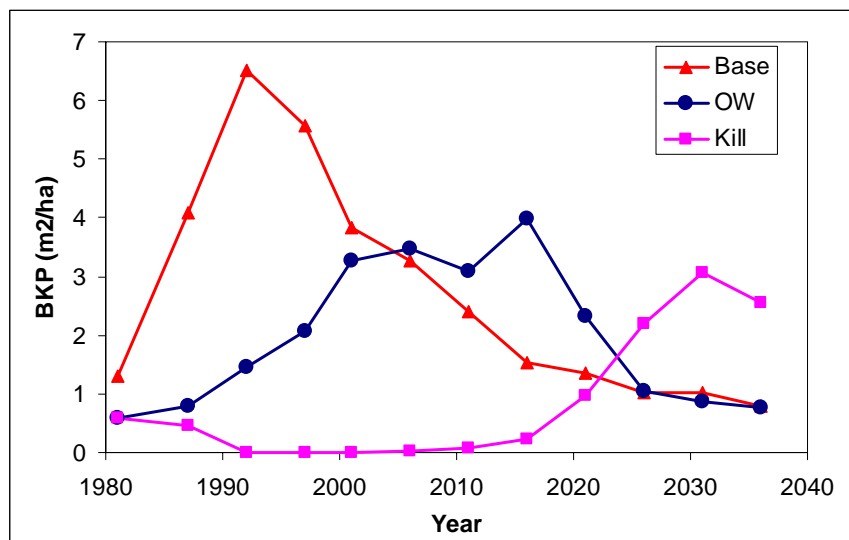


Figure 3.1: Average beetle population in the landscape. BKP is a measure of beetle level related to the size of a tree the beetle could kill. For example, 4 m²/ha of BKP is equivalent to enough beetles to kill 100 trees/ha with 9-12 cm dbh.

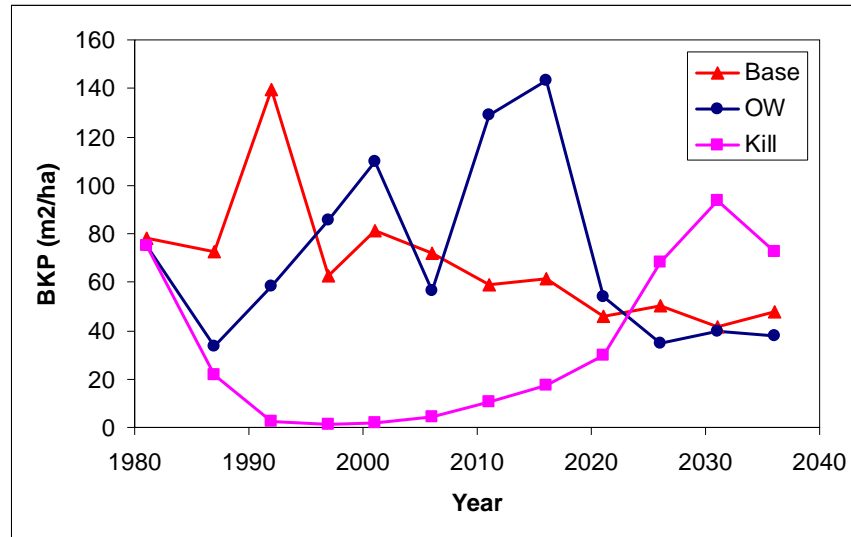


Figure 3.2: Maximum beetle population in any stand the landscape. BKP is a measure of beetle level related to the size of a tree the beetle could kill. For example, 80 m²/ha of BKP is equivalent to enough beetles to kill 1000 trees/ha with 18-21 cm dbh or 2000 trees/ha with 9-12 cm dbh.

The landscape is relatively large, close to 150,000 ha, and the beetles move through the landscape slowly. The graphs of the landscape averages and maximums show how the outbreak is progressing over time, but different parts of the landscape may be experiencing the outbreak at different periods. Again, each scenario shows significantly different patterns of beetle outbreak (Figure 3.3). With no outside world, the beetle epidemic starts near the initial stands and moves outwards slowly through the landscape. By the end of the simulation period, the beetle is concentrated at the far east of the landscape. In the simulation with the outside world, the beetle spreads throughout the landscape more quickly, because beetles can fly in from outside the landscape, and do not have to travel from the initial stands. With the inclusion of the winter kill event, the beetle shows some high mortality near the initial stands, but the majority of the landscape is hit in the last decade of the simulation.

In all scenarios, the *Ips* population was unable to maintain itself past the first five-year timestep. The levels were relatively high in that first timestep, but due to lack of appropriate food (such as slash and stressed trees), or low reproductive rates, the *Ips* did not continue in any stand past that period.

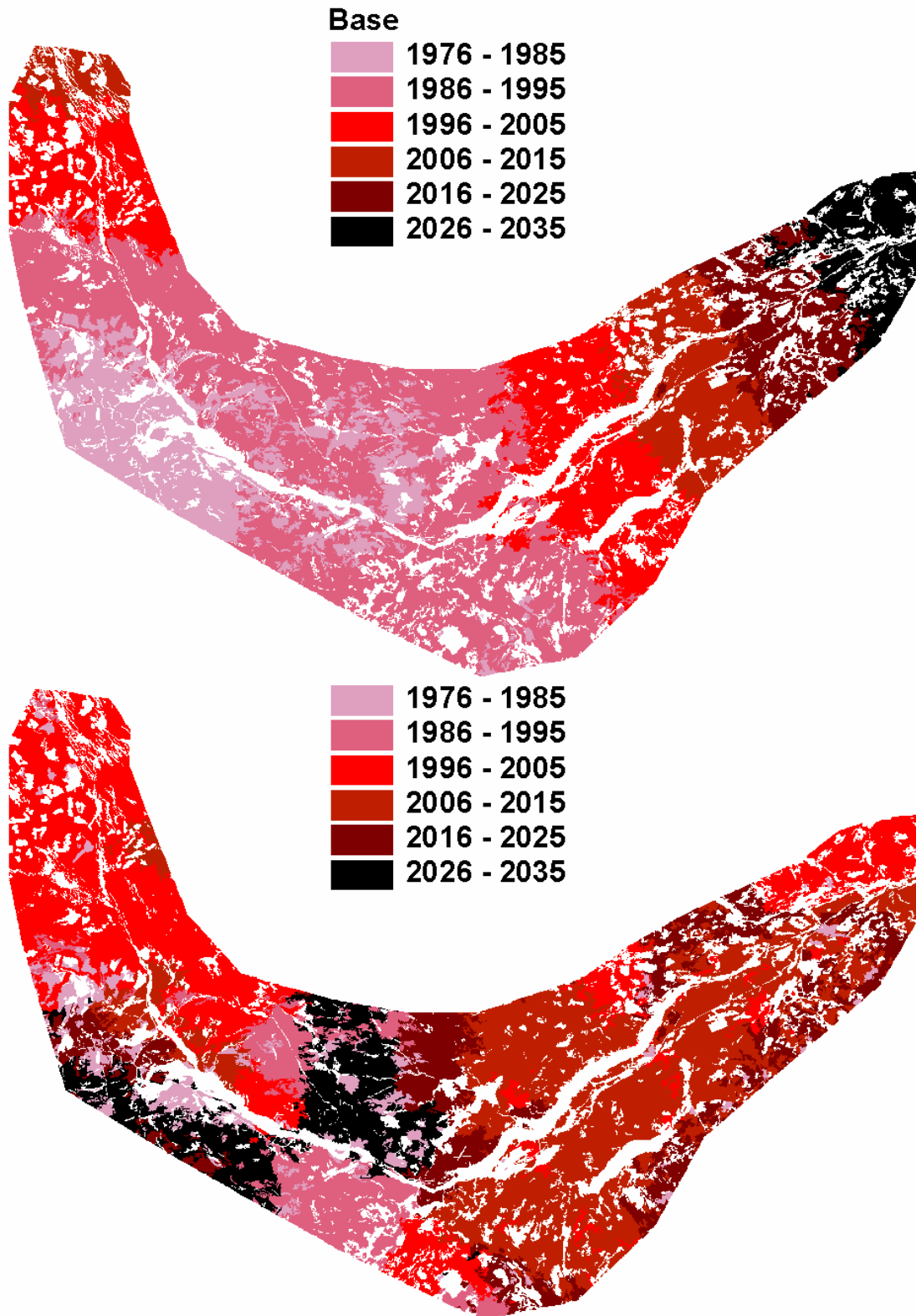


Figure 3.3: Maps showing the year at which the stand experienced its maximum volume kill in each of the three simulations. The shading becomes darker as the decades progress. Top: base scenario (no outside world). Bottom: Outside world.

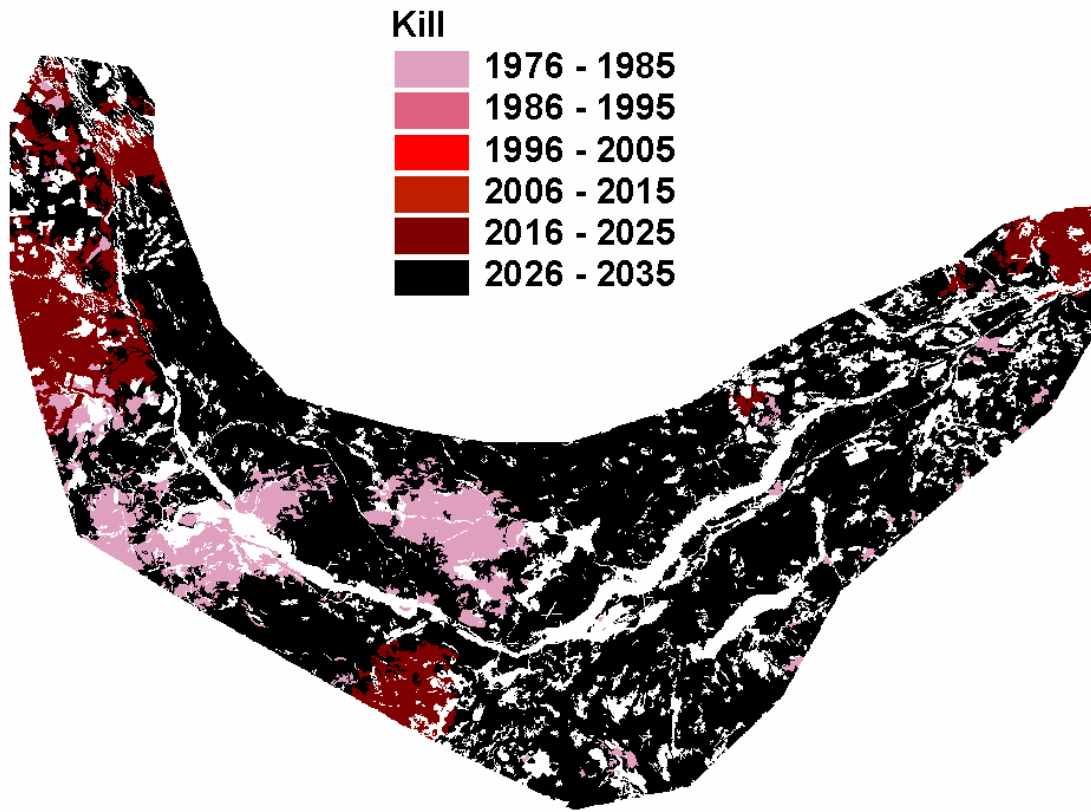


Figure 3.3: (cont.) Kill event (also with an outside world).

Stand-level Results

Eleven of the stands in the landscape were those for which sample data were taken in 1987 and nine of those stands were resampled in 2001. An examination of those stands can give us a better understanding for how well the model behaves as compared to the real world.

One set of indicators is the size distribution of live host trees in the stand. In 1987, the model overpredicts the amount of basal area in the larger size classes compared to the data (Figures 3.4 and 3.5). There are few differences between the scenarios in this year in any stand, mostly because the main beetle mortality in most simulations occurs after 1987.

In 2001, the differences between the data and the simulation results are more pronounced (Figures 3.6 and 3.7). The base scenario generally produces better results than do the other two scenarios. In that simulation, close to 75% of the landscape has already experienced the outbreak. Conversely, for the outside world and kill scenarios, only approximately 33% and 10% of the landscape in each scenario has experienced the outbreak.

Two other factors contribute to the big discrepancy between the data and the simulation results in 2001. In the real world, beetles such as *Ips* are operating on stands in high enough levels to be killing the trees. *Ips* is not well represented in the model, so is not causing any mortality. The second factor which may affect the results is that we assumed the live trees in 1976 were the same size as those in the data in 1987.

This could cause the model to produce larger trees than the data show, even if mortality rates were similar.

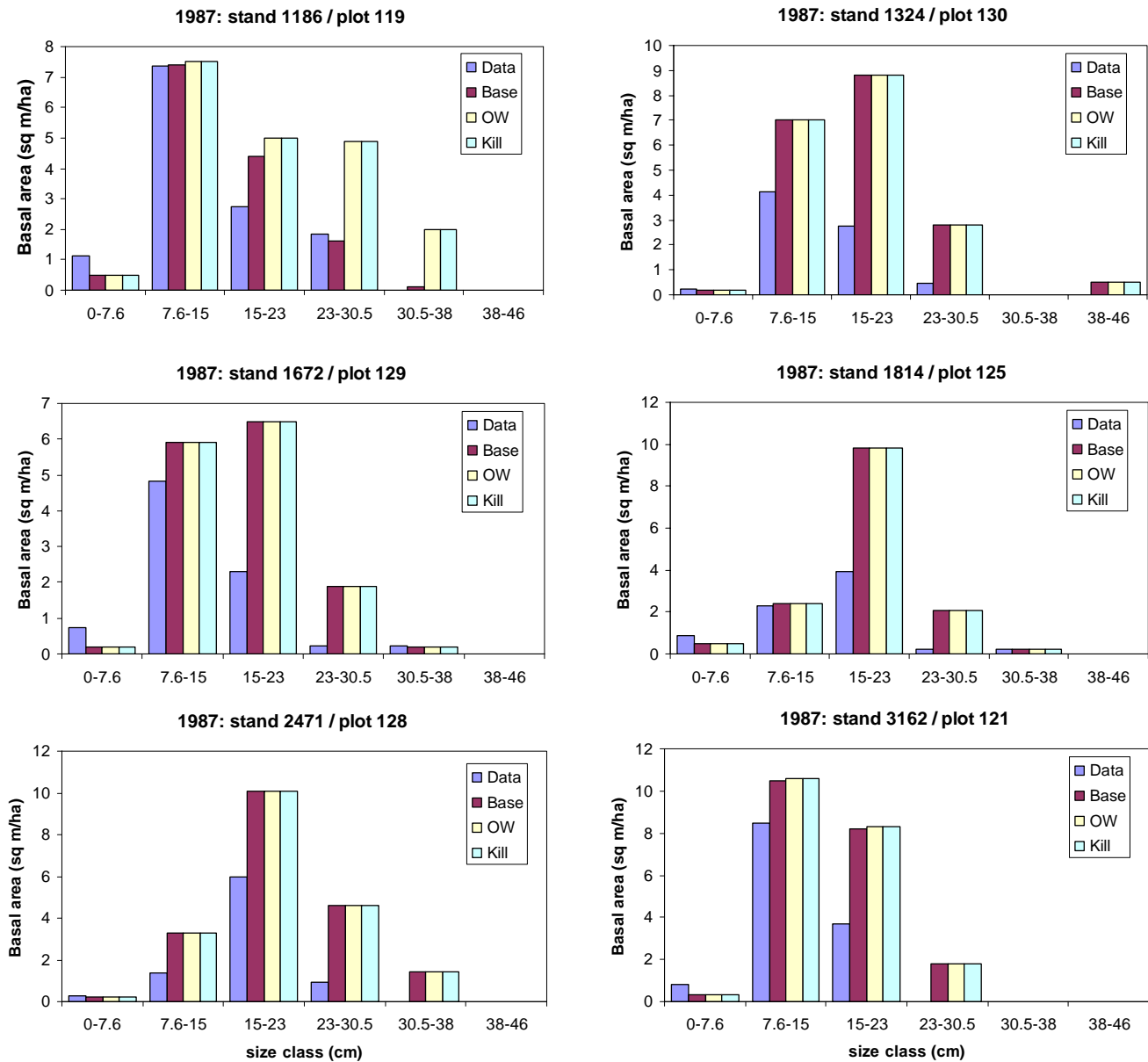


Figure 3.4: Comparison between the 1987 data of the size distribution of host trees in the stand and the results in 1987 for six of the stands. The left-most bar in each case is the data value.

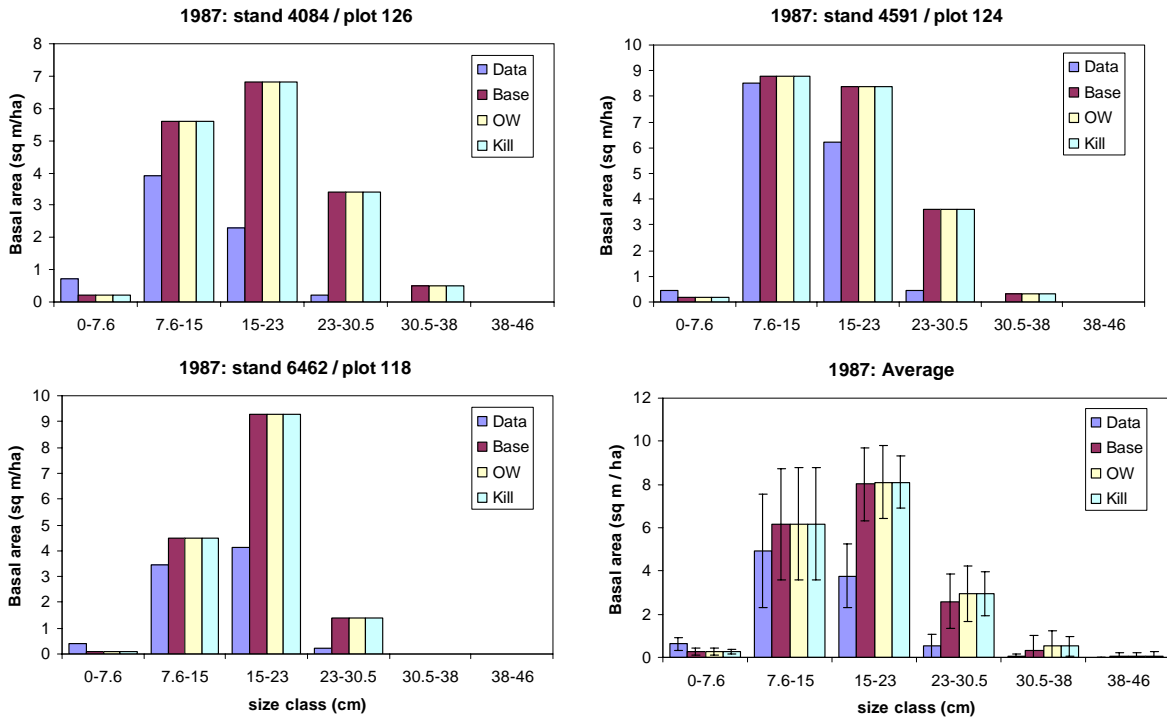


Figure 3.5: Comparison between the 1987 data of the size distribution of host trees in the stand and the results in 1987 for three stands, and the average of all nine stands shown in this figure and Figure 3.4. The left-most bar in each case is the data value.

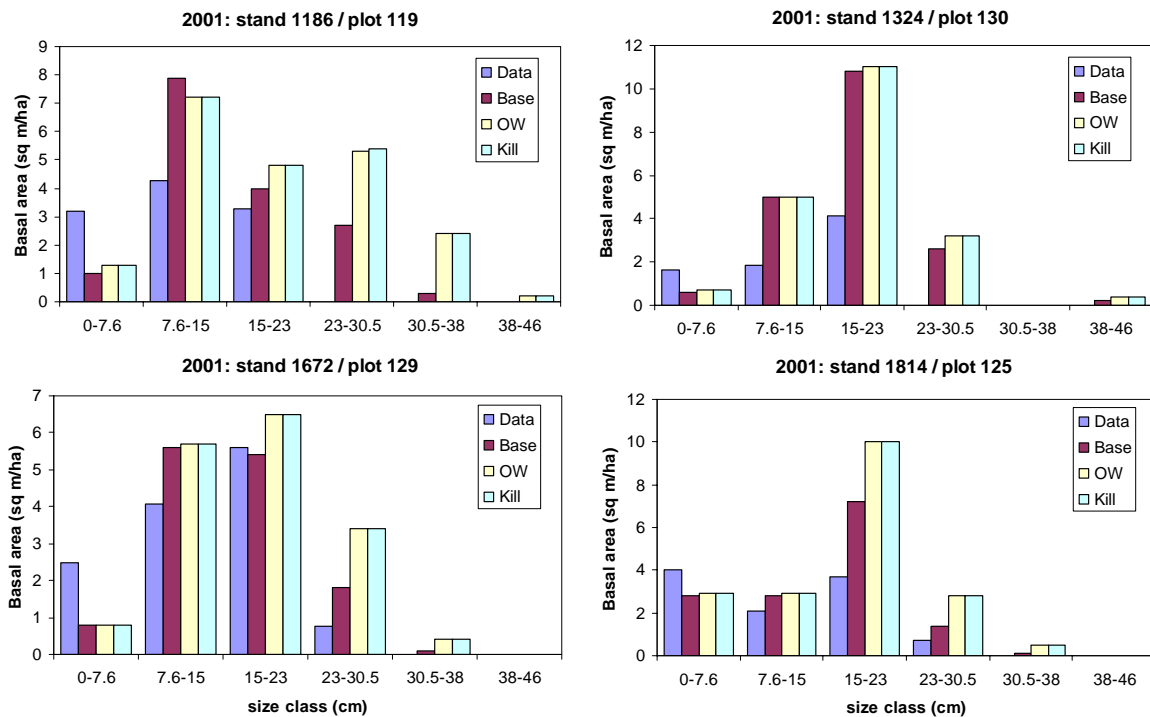


Figure 3.6: Comparison between the 2001 data of the size distribution of host trees in the stand and the results in 2001 for four of the stands. The left-most bar in each case is the data value.

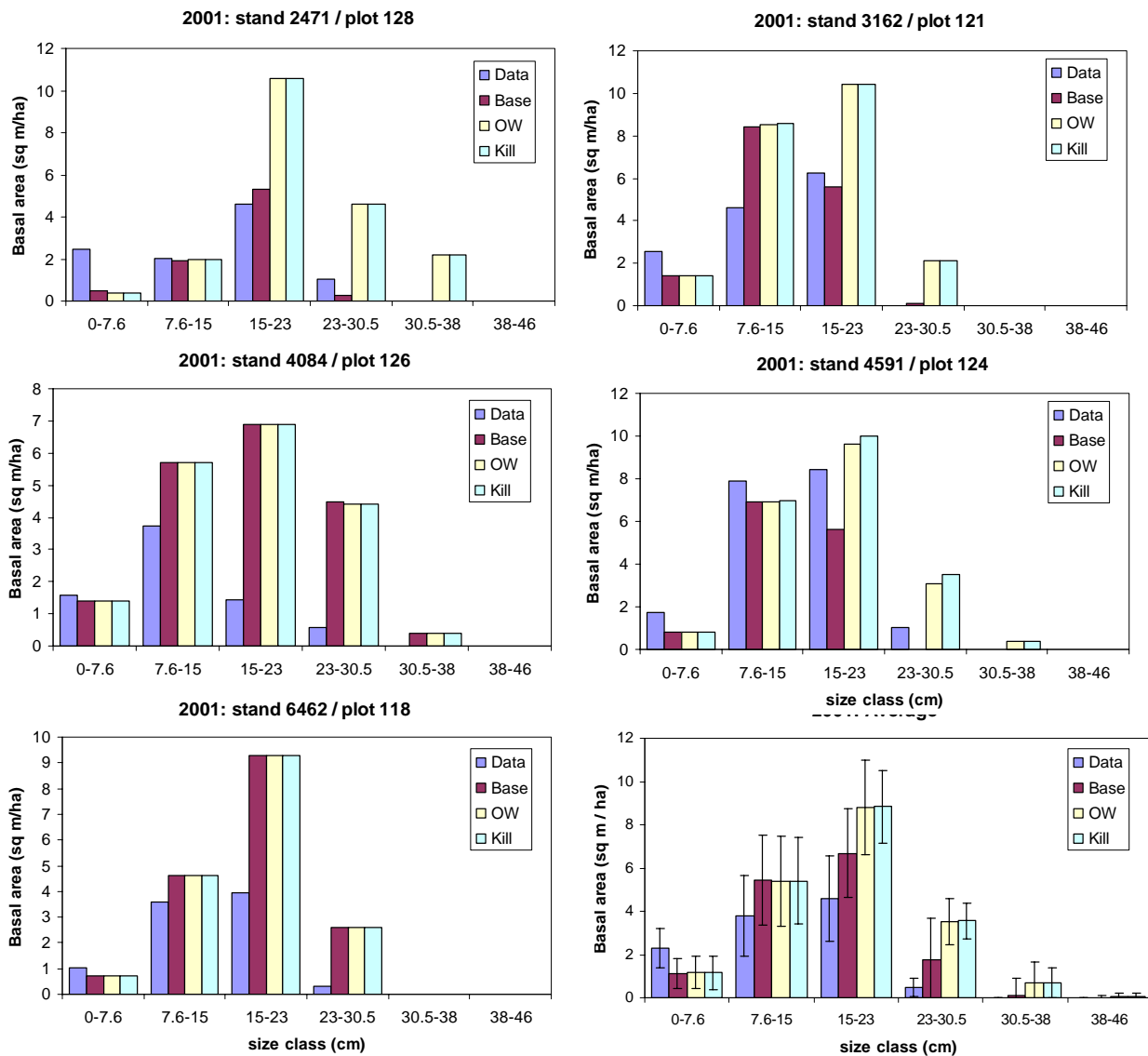


Figure 3.7: Comparison between the 2001 data of the size distribution of host trees in the stand and the results in 2001 for five stands, and the average of all nine stands shown in this figure and Figure 3.6. The left-most bar in each case is the data value

The maps in Figure 3.3 show at a landscape level how the beetle moves through the landscape and how the three scenarios show different patterns. This does not show specifically how different stands might be experiencing the MPB-levels in each scenario. An indicator that does this is the total host volume killed by beetles in each the stand (Figures 3.8). Each stand shows a different pattern. Some, such as stand 1186, experience the outbreak early in the simulation period in all scenarios, whereas others such as stand 4591, have high mortality levels at completely different times in each scenario. Stand 4474 only has high levels of mortality in one scenario. Stands 1672 and 1814 have low levels of mortality in all periods.

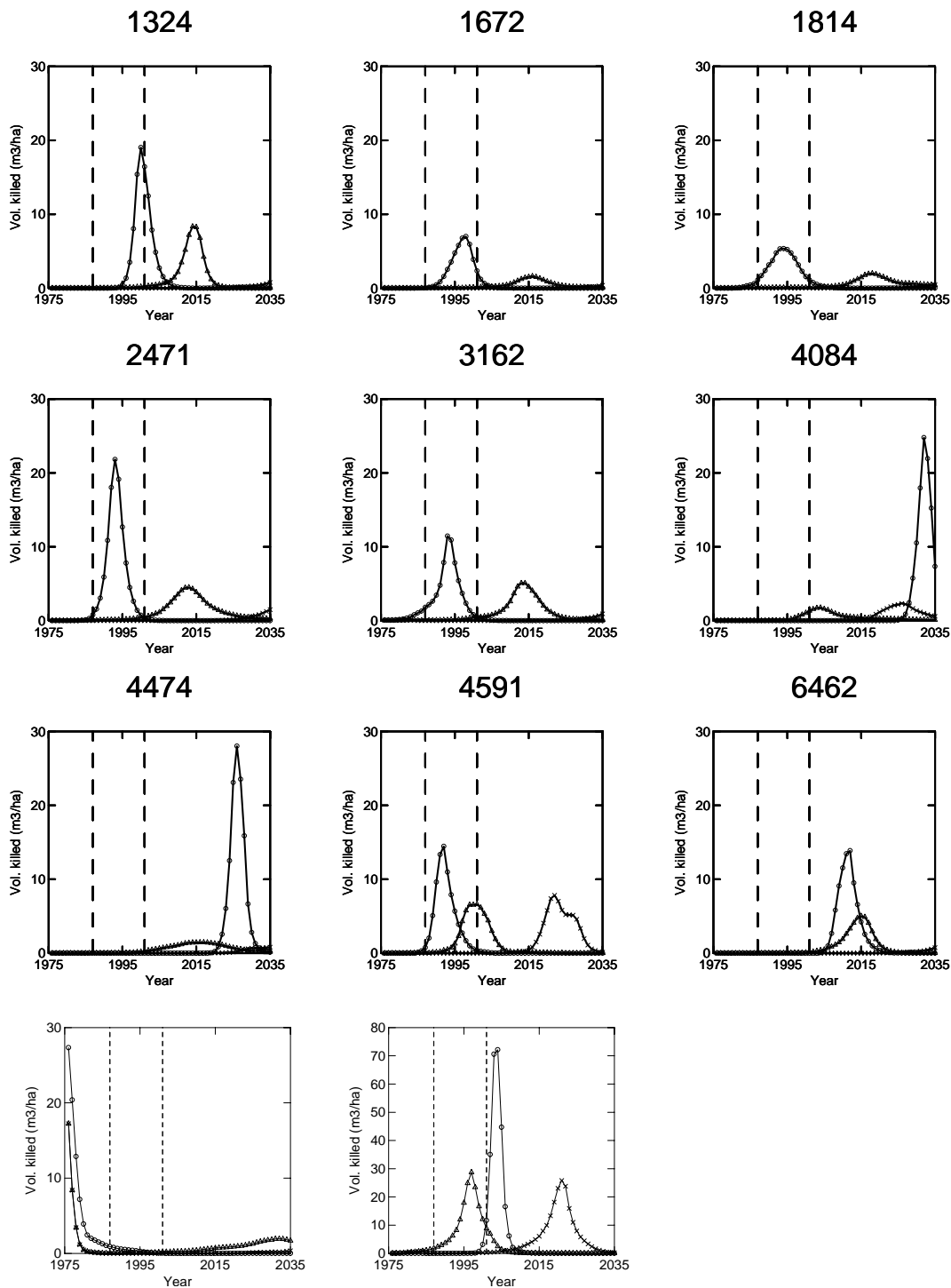


Figure 3.8: Host volume mortality killed by MPB in each scenario in each of the 11 sample stands. Symbols are: Base run: “o”, OW run: “□” and Kill: “x”. Vertical lines show the years 1987 and 2001. Note that the last graph has a different y-axis scale. The last two graphs are stands 1186 and 3767.

4. Conclusions

The WWPBM performed well in terms of demonstrating differences between scenarios, the patterns of MPB dispersal across the landscape, and the ability of the model to show outbreaks in different time periods. The model did not do well, however, in predicting the amount of mortality that occurred in the sample stands, and was not able to reproduce the basal area distributions found in those stands. In addition, *Ips* was not able to maintain itself in the landscape, so a large part of the later mortality that is found in the data from the sample stands cannot be captured.

This study performed some very basic simulations with the prototype linkage of the WWPBM model. We recommend two different types of simulations in order to examine the behaviour of the model in more detail. The first set of recommendations concerns the sensitivity of the model to some of the assumptions that were made that may influence beetle behaviour, but are not directly tied to beetle biology:

1. Revisit how the tree lists were assigned to each stand. There are two ways of doing this. One is to use the same algorithm that is currently being used, but starting with a different random number so that the assignments of tree lists to trees occur differently. The other method would be to use a more rigorous assignment method, such as the Most Similar Neighbour analysis, which uses more stand information such as slope, aspect, and elevation, to ensure that the most appropriate tree list is assigned.
2. Revisit the methods by which the sample information was back-dated to 1976. For example, how many of the dead trees were alive in 1976? What size should the live trees be? The assumption that the live trees were the same size in 1976 as in 1987 will obviously cause the model to over-predict tree sizes. Conversely, however, the low growth multiplier that was used should help mitigate this problem.
3. The maximum basal area that was chosen can have a large impact on model mortality. Simulations with different values could show the impact of this assumption on beetle behaviour. Similarly, we chose to use the same growth multiplier for all stands in the landscape. An alternative approach would have been to use the multiplier only for those stands that did not have growth information. This may have helped influence the choice of beetle towards different stands.

The second set of recommendations is directly related to the beetles.

1. The beetles travelled very slowly through the landscape, due to the fact that we told the model that they could only travel about 4 km. This may have affected their ability to pick the “best” stands and thus outbreak sooner. It would also have limited the number of beetles that could come in from the “outside world” since the size of the outside world depends on the travel distance.
2. *Ips* were unable to support themselves in the landscape. There are several possible changes that could be made to facilitate their ability to survive. We did not change any of the default values in the model for things such as reproduction, travel distance, preference for slash, etc. Since many of these parameters were changed for MPB, it is likely that they will also need to be changed for *Ips*.
3. Review the *Ips* part of the model to try to determine what was limiting *Ips* in the landscape. This may help determine which variables listed above may be the most important for calibration.
4. Review the initialisation of the beetles in the landscape, especially *Ips*.

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